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RESEARCH OF THE STRESSES IN THE LARGE-GRAIN GEARINGS

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Summary: *Problems connected with modeling of the contact interaction of teeth in gearings are considered. The technique of generation rational FE meshes for the solution of such problems is developed. As an example the distribution of stresses straight large-grain gearings is investigated.*
Keywords: *large-grain gearings, finite element method, modeling of contact interaction*

1. INTRODUCTION

Tooth gearings are one of the most widespread elements used in mechanical engineering. It is not surprising, that many works are devoted to them strengthening analysis, there is a plenty of methods of their calculation, and such calculations are standardized in a number of the countries. However the specified calculations until recently used a number of essential simplifications due to which it was possible to carry out such calculations. For example, at calculation of the teeth on the bend durability methods of resistance of materials in which the tooth was replaced with a beam of variable section were used. At calculation on contact durability theory of Hertz in which a number of essential assumptions, for example, absence of friction between surfaces is accepted, replacement real teeth by half-spaces of the set curvature, etc. was used. There were methods of the theory of elasticity, for example, basing on the theory of functions complex variable, in which the real profile of a tooth was replaced with a contour received as a result of it conformal representation. It is obvious, that at such approach enough serious discrepancies were founded already in a target setting. Finite element method widely used now allows to do without the majority of the specified assumptions and, accordingly, to specify the solution of a problem. FEM allows to not divide a problem of the analysis of the stresses of tooth gearings on problems of a bend and contact problems, and to solve a problem in a complex. Its application allows to consider also not only ideal tooth gearings, but also real gearings, taking into account thus deviations of teeth profiles from the nominal sizes, including a consequences of the wear. Discrepancies of assembly of gearing and manufacturing of its separate elements, for example, misalignment, shaft beat, etc. can be taken into account.

2. TARGET SETTING

In considered work the analytical models of the FEM used for the solution of problems of the teeth interaction of the large-grain gearings were investigated. At calculation with using of the FEM one of basic questions is creation of adequate geometrical model, and the main thing, then, is creation of a FE mesh with using of semi-automatic procedure of a mesh generation. Creation of geometrical model begins with a formation of a tooth profile. For these purposes program Teeth in dynamically developing programming language IBASIC has been specially developed. It has allowed at first to receive point-to-point a tooth profile on the basis of the theory of evolvent gearings [1], and then to create suitable for the further export and the analysis to the preprocessor of a package MSC.NASTRAN for Windows - FEMAP a neutral file in format NEU which is an open format of data transmission for the given program. After importation of pinion geometry the gear segment consisting from three зубьев incorporated in one boundary surface is received. Similarly we act for a corresponding cogwheel. Besides it, preliminary, contours of gear segments should be broken by program on the certain parts on which borders at generating of a FE mesh there will be nodes. It is done to avoid the problems connected to an inequality of nodal forces, caused by discrepancy of corresponding nodes, in a contact patch that can result in incorrect results of the calculations. On this step accuracy of results received finally is founded. To carry out splitting it is possible with use of the special curves, named non-uniform curvilinear Bezier splines. Their application allows to use more simple mathematical processing both at a stage of construction, and for the further analysis.

That it will be defined with quantity of elements on a involute part of a contour first of all it is necessary to calculate exact value of length of its both parts. It is possible for making, using mathematical definition of the curve length set parametric, with the help of the define integral

$$s(t) = \int_0^t f(t)dt \tag{1}$$

where *s(t)* – the current length of a curve depending on the current value of parameter *t*, and subintegral function *f(t)* is defined as follows

$$z = f(t) = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ . \tag{2}$$

At definition of the curve length two approaches for the solution of a task in view can be used: an analytical or numerical finding of the integral (1). The first approach can be used after consecutive substitution of reference values in the formula (2), and then in (1). At last, integrating the expression received in result and substituting limits of integration, it is possible to find required length of the involute part of a curve. It is quite possible, as subintegral function - the smooth curve which is not having discontinuities neither of the first, nor the second kind. This approach is enough laborious as after execution of substitutions turning out subintegral function is so difficult, that a finding of integral in quadratures rather doubtfully. Thus risk to commit a technical mistake in final expression it is rather great.

The second variant of the solution of the received problem which has been realized is a numerical method of calculation of value of integral for a preset value of parameter, for example t^*. In this case the site of integration from *0* up to t^* shares on 100 equal parts, the length of each of which is equal Δ, and further value of integral (1) is calculated on a method of trapezes. The final formula for calculation of the general area limited to a curve $z = f(t)$, i.e. required length a involute curve, is possible to present as

$$s\left(t^*\right) = \int_0^{t^*} f(t)dt = \Delta\left(\frac{z_0}{2} + z_1 + \ldots + z_{99} + \frac{z_{100}}{2}\right) \tag{3}$$

It is obvious, that in this case at $t^* = 1$ length of an arch, for example, for top involute part $\cup S_2$ will be equal $s(1) = S_2$.

In preprocessor FEMAP for the indication of control "nodal" points serves so-called Bias Factor. It is the parameter specifying in how many time the characteristic of a final arch exceeds the characteristic initial. Here the characteristic is meant concept a way of the assignment of nodal points. This assignment can be depending on the current parameter *t* or depending on the current length of an arch. In a considered case preferably the assignment of splitting of curves depending on the current length of an arch as parametrical dependence $x(t)$ has square-law character. It means, for example, that at change of parameter with equal step from *0* up to *1*, the curve will not be divided into equal parts on length that is critically necessary.

The choice only Bias Factor cannot unambiguously define the future position of the nodes along any curve. It is necessary choose what sort dependence between lengths of parts should to be. Division of a curve into equal parts is possible, in this case in field Node Spacing the parameter "Equal" gets out. Lengths of consistently going parts can make an arithmetic progression, the parameter "Biased" in this case gets out. In the same dialogue it is necessary to set, where exactly will smallest elements settle down (in case of non-uniform arrangement of nodes) on the beginning, on the end, on the middle of a curve, or on its both ends. In this dialogue (at the manual assignment of the FE mesh) the quantity of elements, or their maximal size is indicated also.

For splitting involute parts of tooth profiles on separate elements it is necessary to determine amount of elements on the top and bottom parts of a profile. Thus for program realization of described algorithm it was necessary to set first of all total of elements on involute part of a profile N_S, as the sums of number of elements on the bottom and top parts of a profile, accordingly, N_1 and N_2. Initially we believe, that it was possible to distribute in regular intervals elements along a profile of a tooth. Then the final formula for definition N_2 looks as

$$N_2 = \frac{S_2}{S_1 + S_2} N_S\ . \tag{4}$$

From this formula becomes obvious, that, unfortunately, calculated value N_2 most likely will not be the integer. In the program this value is approximated to the nearest integer. The quantity of elements for the bottom part of involute N_1 can be accordingly determined. Now, when lengths of both parts of a profile and quantity of elements on each part of the involute curve are known, Bias Factor for each of parts of a profile can be calculated. In the right part of figure 1 there is schematically represented arrangement of required control points in which nodes of a finite element mesh will be formed. Let control points are placed in such a manner that lengths of sites (the future elements) form an arithmetic progression, increasing to top of a tooth. Apparently from figure, the control point with number 0 coincides with a point of profile B and is a reference point of length of a profile arch. The point with number 1 is the end of the first part which length is equal s_1. Thus, s_1 - the first member of an arithmetic progression. The following part of a profile from a point 1 up to a point 2 will have length equal $s_1 + \Delta$, where Δ - a difference of an arithmetic progression. For simplification of recognition in tab. 1 lengths of separate part, and also lengths of arches up to corresponding control points are resulted.

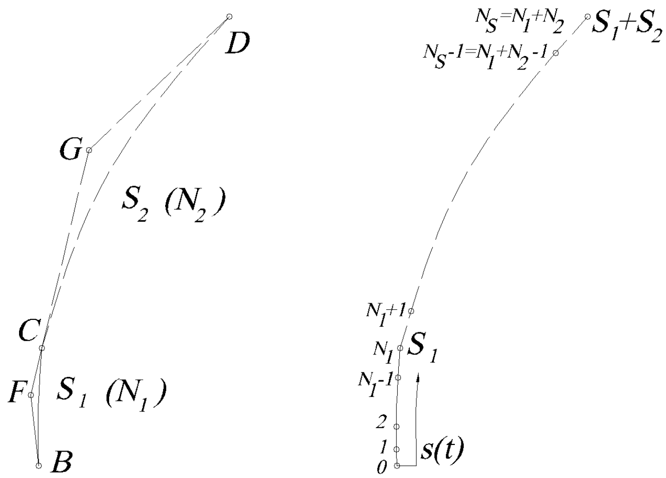

Figure 1: Arrangement of control points along involute curve

On the basis before the entered definition, that Bias Factor is the parameter indicating in how many time length of a final part exceeds length initial, it is necessary to define size of these parts. On fig. 1 it is pieces between points *1,2* and $N_1 - 1$, N_1, and also N_1, $N_1 + 1$ and $N_S - 1$, N_S for the bottom and top part of involute accordingly. For their definition it is necessary to calculate before a difference and the first member of an arithmetic progression that it is possible to make as follows. On the basis of the tabulared data it is necessary to write down system from two equations, where in the first equation will be written down expression for a finding of length of the bottom part of the involute structure (from a point *0* up to N_1), and in the second - for the general length of the involute curve.

$$\begin{cases} N_1 s_1 + \dfrac{N_1(N_1 - 1)}{2}\Delta = S_1 \\ N_S s_1 + \dfrac{N_S(N_S - 1)}{2}\Delta = S_1 + S_2 \end{cases} \tag{5}$$

Considering the equations (5) as system for a finding of unknown s_1 и Δ, it is possible to find a difference of arithmetic progression Δ and its first member

$$\Delta = \frac{2}{N_1 N_S} \cdot \frac{S_1 N_S - (S_1 + S_2) N_1}{N_1 - N_S}, \tag{6}$$

$$s_1 = \frac{1}{N_1}\left[S_1 - \frac{N_1(N_1 - 1)}{2}\Delta\right]. \tag{7}$$

Now finding of required values is possible. Bias Factors for the bottom and top parts of the involute curve are equal accordingly

$$b_{S1} = \frac{s_1 + (N_1 - 1)\Delta}{s_1}, \qquad b_{S2} = \frac{s_1 + (N_S - 1)\Delta}{s_1 + N_1 \Delta}. \tag{8}$$

Table 1: To the explanatory of arrangement of the control points

Number of a point N	General length of a curve from a zero point $s(t)$	The current length of a piece $\Delta s(t)$
0	0	
1	s_1	s_1
2	$s_1 + \Delta s_2 = 2s_1 + \Delta$	$s_1 + \Delta$
3	$s_2 + \Delta s_3 = 3s_1 + (1+2)\Delta$	$s_1 + 2\Delta$
4	$s_3 + \Delta s_4 = 4s_1 + (1+2+3)\Delta$	$s_1 + 3\Delta$
...	...	...
$N_1 - 1$	$s_{N1-2} + \Delta s_{N1-1} = (N_1 - 1)s_1 + [1+2+...+(N_1-2)]\Delta$	
N_1	$s_{N1-1} + \Delta s_{N1} = N_1 s_1 + [1+2+...+(N_1-1)]\Delta = S_1$	$s_1 + (N_1 - 1)\Delta$
$N_1 + 1$	$s_{N1} + \Delta s_{N1+1} = (N_1 + 1)s_1 + [1+2+...+N_1]\Delta$	$s_1 + N_1\Delta$
...	...	...
$N_S - 1$	$s_{NS-2} + \Delta s_{NS-1} = (N_S - 1)s_1 + [1+2+...+(N_S-2)]\Delta$	
N_S	$s_{NS-1} + \Delta s_{NS} = N_S s_1 + [1+2+...+(N_S-1)]\Delta = S_1 + S_2$	$s_1 + (N_S - 1)\Delta$

3. RESULTS OF CALCULATIONS AND CONCLUSIONS

The received information is sufficient to realize discretization of the involute profile of a tooth. Such discretization for other parts of a gear segment does not represent additional complexities and can be executed similarly. Being guided by the description of a format of the neutral (neu) file, which could be found in the open documentation for preprocessor/postprocessor FEMAP, the block of the program has been made, which has allowed to write down all necessary parameters in a final file of results. Coordinates of all points, including control and basic, their numbers, for curves - color, type, quantity of elements along a curve, Bias Factors and set of other technical information, and also definition of a surface and number of curves of which it will consist are written in a file. After processing an available neutral file in FEMAP and generation of the FE mesh it is necessary to make export received FE models in MSC.MARC and further to carry out calculation. On fig. 2 the characteristic example of results of calculation is shown.

For research of the dynamics of a drive the magnitude of the gearing rigidity which is a variable has the big value. The developed technique has allowed to define dependence of the gearing rigidity on a corner of turn (a relative arrangement of cogwheels). Calculations were carried out for various tooth gearings, among which large-sized tooth gearings of miner mills or gearings of traction drives of electric locomotives.

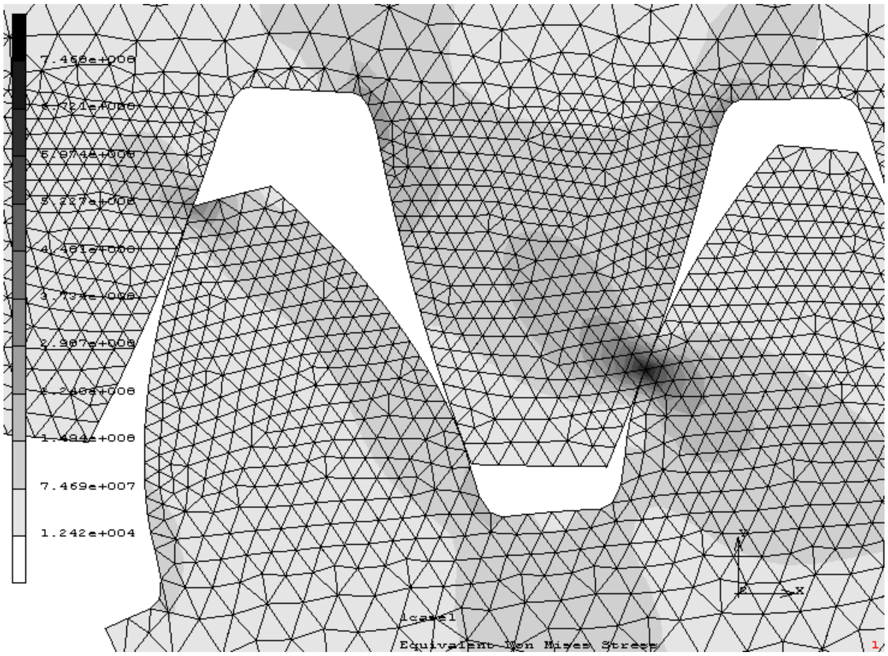

Figure 2: Distribution of equivalent VonMises stress at contact interaction of teeth

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